Journal of Applied Mechanics and Technical Physics, Vol. 41, No. 3, 2000

CIRCULATION FLOW AROUND AIRFOILS BY A STEADY PLANE-PARALLEL FLOW OF A HEAVY LIQUID OF FINITE DEPTH WITH A FREE SURFACE

K. E. Afanas'ev and S. V. Stukolov

UDC 532.5

Steady problems of a circulation flow around bodies by a flow of a heavy liquid bounded by a free surface and a straight bottom are solved. The method of complex boundary elements is used, which is based on the integral Cauchy formula written for a complex-conjugate velocity. Results of numerical calculations of the flow around a circular contour and the Joukowski airfoil are presented. Shapes of the free surface and the most important hydrodynamic characteristics of the process (velocity circulation over the airfoil and the lifting force and its moment relative to the sharp edge of the airfoil) are given.

A great number of papers are devoted to solving problems of the flow around airfoils by different methods. Belotserkovskii et al. [1] employ the method of discrete vortices, which is widely used in calculations.

The solution of boundary-value problems of the flow around airfoils often reduces to the solution of singular integral equations, which have a parametric singularity (maximum thickness of the airfoil) in addition to the singular one. This singularity is manifested in the fact that the distance between the neighboring points on the upper and lower sides of the contour decreases as the airfoil becomes thinner. As a result, equations written separately for the upper and lower surfaces of the airfoil become identical, which creates many difficulties in numerical solution of the problem for the case of thin airfoils and in the vicinity of the trailing edge.

Gorelov [2] obtained a system of integral equations in shear components of velocities at the upper and lower airfoil surfaces, which does not possess this parametric singularity. The system of equations is solved by the method of discrete vortices, and it is shown that this method is applicable for solving problems of the flow around airfoils of an arbitrary thickness, including the infinitesimal one.

Using the method of boundary elements (MBE), Terent'ev and Kartuzova [3] investigated a circulation flow around a system of airfoils by an unbounded liquid flow. The third Green formula for the stream function is used as an integral relation.

The MBE with the third Green formula for the velocity-potential function $\varphi(x, y)$ cannot be used to solve problems of the circulation flow around airfoils, since $\varphi(x, y)$ is not uniquely determined and experiences the first-kind discontinuity in the sharp edge of the airfoil for circulation other than zero, whereas the stream function $\psi(x, y)$ remains continuous [4]. Nevertheless, if we introduce a new single-valued function $\Phi(x, y) = \varphi(x, y) - \Gamma \arg z/(2\pi)$, we can use the third Green formula in the solution, and the circulation Γ enters the solution as an additional unknown.

Yas'ko [5] considered a steady problem of the flow around airfoils using the MBE with the third Green formula for the stream function in a bounded flow with a free surface. To determine the unknown free boundary, Yas'ko proposed iteration algorithms for high and low Froude numbers, but for Froude numbers close to unity, the solution cannot be obtained using the method described in [5].

Kemerovo State University, Kemerovo 650043. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 41, No. 3, pp. 101–110, May–June, 2000. Original article submitted June 2, 1999; revision submitted July 20, 1999.

 0021-8944/00/4103-0470 $25.00

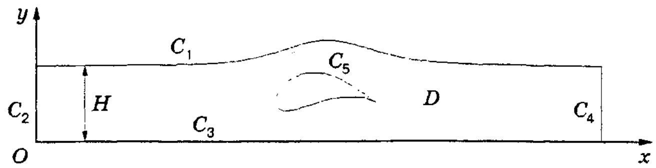

Fig. 1

Mokry [6] proposed a method for studying the flow around an airfoil, which employs the integral Cauchy formula written for the shear and normal components of velocity. Solutions are obtained both for an isolated airfoil and for a system of airfoils in an unbounded flow.

A modified complex method of boundary elements (CMBE) is proposed in the present paper to solve steady problems of the circulation flow around airfoils by a liquid flow of finite depth with a free surface. The accuracy of the method and the algorithm for constructing the free boundary was determined in test calculations.

1. Formulation of the Problem. Let an isolated airfoil with boundary C_5 be located in a flow of a ponderable liquid bounded by the free surface C_1, straight bottom C_3, and inflow and outflow sections C_2 and C_4, respectively (Fig. 1). We introduce the following notation: the inflow velocity V_∞ and the flow depth H. We consider the problem in the plane of the complex variable $z = x + iy$. The motion of the liquid is described by the function $w(z) = \varphi(x, y) + \psi(x, y)$, where φ is the velocity potential and ψ is the stream function. In the circulation flow around the airfoil, the potential has a discontinuity whereas the velocity field remains continuous; therefore, the problem can be more easily solved in terms of the complex-conjugate velocity:

$$W(z) = \frac{\partial\varphi}{\partial x} + i\,\frac{\partial\psi}{\partial x} = \frac{\partial\varphi}{\partial x} - i\,\frac{\partial\varphi}{\partial y} = V_x - iV_y. \tag{1}$$

Let ξ be a point that belongs to the contour; then expression (1) can be written in the form

$$W(\xi) = V_x(\xi) - iV_y(\xi) = (V_n(\xi) - iV_s(\xi))\,\mathrm{e}^{-i\alpha(\xi)},$$

where V_s and V_n are the shear and normal components of velocity at the point ξ and α is the angle between the direction of V_n and the Ox axis.

The problem of the flow around an airfoil can be reduced to solving the Laplace equation

$$\Delta W(z) = 0, \qquad z \in D$$

for the function $W(z)$, which is analytical in the flow domain D. The no-slip condition is satisfied on the airfoil and on the bottom: $V_n = 0$ ($z \in C_5, C_3$). The conditions of liquid inflow and outflow are imposed at the side boundaries: $V_n = \mp V_\infty$ ($z \in C_2, C_4$).

Let V_s^+ and V_s^- be the shear components of velocity vectors when approaching the sharp edge from the upper and lower sides of the airfoil, respectively. Then the Joukowski–Chaplygin condition can be written as $V_s^+ + V_s^- = 0$. Introducing the notation $V_s^0 = V_s^+ + V_s^-$, we write the Joukowski–Chaplygin condition in the form

$$V_s^0 = 0. \tag{2}$$

In this case, the angle α is formed by the bisectrix of the corner in the sharp edge and the Ox axis.

The free boundary is a streamline on which the Bernoulli equation is valid:

$$|W(z)|^2 = 1 - 2(\mathrm{Im}\, z - 1)/\mathrm{Fr}^2 \qquad (z \in C_1). \tag{3}$$

Here $\mathrm{Fr} = V_\infty/\sqrt{gH}$ (g is the acceleration of gravity). The free surface C_1 is unknown *a priori* and should be found numerically in the course of solving the problem.

2. Numerical Method. For the function $W(z) = V_x(x,y) - iV_y(x,y)$, which is analytical in the domain D and limited by the piecewise-smooth boundary $C = \bigcup_{j=1}^{5} C_j$, the integral Cauchy formula is valid, which can be written in the following form using Sokhotskii's limiting formulas:

$$W(z_0) = \frac{1}{\varepsilon(z_0)i} \int_C \frac{w(z)}{z - z_0}\, dz. \tag{4}$$

Here we have $\varepsilon(z_0) = 2\pi$ for an internal point, $\varepsilon(z_0) = \pi$ for a point on the smooth boundary C, and $\varepsilon(z_0) = \theta$ for a corner point of the boundary C (θ is the apex angle). The positive direction of motion along the contour C is taken such that the domain D remains on the left (anticlockwise motion along the external boundary and clockwise motion along the internal boundary).

Since V_s and V_n on the boundaries of the domain are known in the course of solving the problem, Eq. (4) should be rewritten as

$$W(z_0) = (V_n(z_0) - iV_s(z_0))\,\mathrm{e}^{-i\alpha(z_0)} = \frac{1}{\varepsilon(z_0)i} \int_C \frac{(V_n(z) - iV_s(z))\,\mathrm{e}^{-i\alpha(z)}}{z - z_0}\, dz.$$

Knowing V_s and V_n, we can find V_x and V_y using the formulas

$$V_x = V_n \cos\alpha - V_s \sin\alpha, \qquad V_y = V_s \cos\alpha + V_n \sin\alpha. \tag{5}$$

Since the shear component of velocity on the free surface and the normal component of velocity on the bottom, side walls, and airfoil are known, we obtain a mixed boundary-value problem for the function $W(z)$. A numerical solution of this problem can be obtained by dividing the contour C into N linear elements Γ_j by the nodes z_j ($j = \overline{1, N}$). Then we have $W(z) = \lim\limits_{\max|\Gamma_j| \to 0} G(z)$, where $G(z) = \sum_{j=1}^{n} W_j \Lambda_j(z)$ is the global linear test function for $z \in \sum_{j=1}^{n} \Gamma_j$. W_j is the value of $W(z)$ at the point z_j, and $\Lambda_j(z)$ is the linear basis function:

$$\Lambda_j(z) = \begin{cases} (z - z_j)/(z_j - z_{j-1}), & z \in \Gamma_{j-1}, \\ (z_{j+1} - z)/(z_{j+1} - z_j), & z \in \Gamma_j, \\ 0, & z \notin \Gamma_{j-1} \cup \Gamma_j. \end{cases}$$

After this division and linear approximation of the function $W(z)$ at the boundary, the Cauchy integral can be calculated analytically in the sense of the main value as $z \to z_j$. As a result, we obtain

$$2\pi i W_j = W_{j+1} - W_{j-1} + W_j \ln \frac{z_{j+1} - z_j}{z_{j-1} - z_j} + \sum_{\substack{m=1,\\ m \neq j, j+1}}^{N} I_m, \tag{6}$$

where

$$I_m = W_{m+1} - W_m + \Big[\frac{(z_j - z_m)W_{m+1}}{z_{m+1} - z_m} - \frac{(z_j - z_{m+1})W_m}{z_{m+1} - z_m}\Big] \ln \frac{z_{m+1} - z_j}{z_m - z_j}.$$

Substituting the decomposition of the complex velocity into shear and normal components into (6), we obtain

$$2\pi i (V_{n_j} - iV_{s_j})\,\mathrm{e}^{-i\alpha_j} = (V_{n_{j+1}} - iV_{s_{j+1}})\,\mathrm{e}^{-i\alpha_{j+1}}$$

$$- (V_{n_{j-1}} - iV_{s_{j-1}})\,\mathrm{e}^{-i\alpha_{j-1}} + (V_{n_j} - iV_{s_j})\,\mathrm{e}^{-i\alpha_j} \ln \frac{z_{j+1} - z_j}{z_{j-1} - z_j} + \sum_{\substack{m=1\\ m \neq j, j+1}}^{N} I_m, \tag{7}$$

where

$$I_m = (V_{n_{m+1}} - iV_{s_{m+1}})\,\mathrm{e}^{-i\alpha_{m+1}} - (V_{n_m} - iV_{s_m})\,\mathrm{e}^{-i\alpha_m}$$

$$+\left[\frac{(z_j - z_m)(V_{n_{m+1}} - iV_{s_{m+1}})\,\mathrm{e}^{-i\alpha_{m+1}}}{z_{m+1} - z_m} - \frac{(z_j - z_{m+1})(V_{n_m} - iV_{s_m})\,\mathrm{e}^{-i\alpha_m}}{z_{m+1} - z_m}\right] \ln \frac{z_{m+1} - z_j}{z_m - z_j},$$

$$\mathrm{e}^{-i\alpha_j} = -i\Big(\frac{z_{j+1} - z_j}{|z_{j+1} - z_j|^2} + \frac{z_j - z_{j-1}}{|z_j - z_{j-1}|^2}\Big)\Big/\Big|\frac{z_{j+1} - z_j}{|z_{j+1} - z_j|^2} + \frac{z_j - z_{j-1}}{|z_j - z_{j-1}|^2}\Big|.$$

Writing Eq. (7) for each boundary node and separating the imaginary and real parts, we obtain

$$SX + iBX = 0,$$

where S and B are the completely filled matrices $N \times 2N$ (N refers to rows and $2N$ to columns) and $\boldsymbol{X} = \boldsymbol{X}(V_{s_1}, V_{n_1}, V_{s_2}, V_{n_2}, \ldots, V_{s_N}, V_{n_N})$ is a vector. Following [7], we obtain the system of equations

$$Q\boldsymbol{X} = \boldsymbol{F}, \tag{8}$$

where the matrix Q and the right-side vector $\boldsymbol{F}$ are obtained as follows. If the shear component of velocity V_{s_j} is prescribed at the node z_j, the jth row of the matrix B is taken. After choosing the elements of the row corresponding to unknown values of V_s or V_n at all other nodes, we obtain the jth row of the matrix Q, the jth element of the vector $\boldsymbol{X}$ corresponds to V_{n_j}, and the jth element of the vector $\boldsymbol{F}$ is the sum of the known values of V_s or V_n multiplied by the corresponding elements of the matrix B. If the normal component of velocity V_{n_j} is specified in the node z_j, the matrix S is used to construct system (8).

System (8) is solved using the Gauss method along the leading element.

3. Algorithm for Constructing the Free Boundary. *Determination of* V_s. It should be noted that the steady problem of the flow around an obstacle has a nonunique solution for a certain range of Froude numbers. The qualitative behavior of this problem can be determined by an example of the flow without an obstacle. The trivial solution of this problem for arbitrary Froude numbers is a uniform flow, and the other solution is a solitary wave. It is shown [8, 9] that this problem has a unique solution if we use the quantity $V = V_0/V_\infty$, where V_0 is the velocity at the wave crest, as the governing parameter instead of the Froude number. For the parameter V, Eq. (3) can be written in the form

$$|W| = \sqrt{1 - (1 - V^2)(y - 1)/(y_0 - 1)},$$

where y_0 is the ordinate of the point of the free surface in which the velocity V_0 is set.

Since the boundary C_1 is a streamline, the velocity vector on it is directed tangentially to the contour. Hence, it follows that $|W| = V_s$. For all points of the free boundary, we have

$$V_{s_j} = \sqrt{1 - (1 - V^2)(y_j - 1)/(y_0 - 1)}, \tag{9}$$

where $j = \overline{1, N_g}$ are the numbers of node points of the free boundary.

Determination of the Shape of the Free Boundary. Let a certain position of the free boundary $C_1^{(k)}$ be known. The algorithm for finding the free boundary has the following stages:

1) the values of V_{s_j} in the nodes z_j on $C_1^{(k)}$ are found from Eq. (9);

2) the system of linear equations (8) is solved;

3) the values of the velocity-vector components V_{x_j} and V_{y_j} are determined at the points of the free boundary $C_1^{(k)}$ using (5);

4) from the condition of collinearity of the velocity vector and the tangent to the boundary $dy/dx = V_y/V_x$, a new position of the free boundary $C_1^{(k+1)}$ is calculated by the formula

$$y_{j+1}^{k+1} = y_j^{k+1} + \Delta y_j^k,$$

where the increment Δy_j^k is determined on the basis of expansion into a Taylor series:

$$\Delta y_j^k = \frac{V_{y_j}}{V_{x_j}}(x_{j+1} - x_j) + \frac{1}{2!}\frac{d}{dx}\Big(\frac{V_{y_j}}{V_{x_j}}\Big)(x_{j+1} - x_j)^2 + \ldots + \frac{1}{4!}\frac{d^3}{dx^3}\Big(\frac{V_{y_j}}{V_{x_j}}\Big)(x_{j+1} - x_j)^4.$$

The cycle is repeated until the required accuracy $\max\limits_j |y_j^{k+1} - y_j^k| < \varepsilon$ is reached, and then the Froude number is calculated using the formula $\mathrm{Fr} = \sqrt{2(y_0 - 1)/(1 - V^2)}$. The straight line $y^0 = 1$ is used as the

TABLE 1

N	E_1	$K(Q)$
77	$1.2 \cdot 10^{-2}$	9.44
144	$2.8 \cdot 10^{-3}$	10.32
288	$6.9 \cdot 10^{-4}$	11.26
496	$1.7 \cdot 10^{-4}$	13.43

TABLE 2

	Present work		[9]	
V	Fr	A	Fr	A
0.01	1.2913	0.8337	1.2909	0.8332
0.70	1.1541	0.3396	1.1540	0.3395

zero approximation; an exception is the vicinity of the point y_0 where the initial value is assumed to be $y_0^0 = 1 + 0.001$.

4. Test Calculations. The accuracy of the modified CMBE was checked using the following test. We have to find a solution of the Laplace equation for the function $W(z)$ in the domain $D = \{0 \leqslant x \leqslant 2\pi;\ -1 \leqslant y \leqslant 0.5 \sin x\}$, in which the no-slip condition $V_n = 0$ is imposed on the bottom and vertical walls and the harmonic function $V_s = (\sin x \cosh(y+1) - \cos^2 x \sinh(y+1))/\sqrt{1+\cos^2 x}$ is set at the upper boundary. Table 1 (column 2) shows the relative error $E_1 = \max|V_n^{\text{exact}} - V_n^{\text{num}}|/\max|V_n^{\text{exact}}|$ depending on the number of boundary nodes, where V_n^{num} is the numerical value of the function obtained by the CMBE and $V_n^{\text{exact}} = (\sin x \cosh(y+1) + \sinh(y+1))\cos x/\sqrt{1+\cos^2 x}$ is the exact value. The conditionality numbers $K(Q)$ of system (8) are listed in column 3.

The objective of the second test was to check the algorithm of constructing the free surface. In the absence of an obstacle in the flow, by varying the parameter V, we obtained solitary steady waves whose parameters were little different from those obtained by Mokry [6], who used a similar method for constructing the free boundary, but the solution is based on the conventional CMBE. The Froude numbers Fr and amplitudes A of solitary waves are listed in Table 2. The values obtained using the modified CMBE and traditional CMBE (the results are borrowed from [9]) are listed in columns 2–3 and 4–5, respectively.

The third test was performed on the problem of the flow around the Joukowski airfoil by an unbounded liquid flow. This problem has an analytical solution and is a good test verification of the numerical result. The Joukowski airfoil can be prescribed parametrically in the form [3]

$$x(t) = \frac{c(c^2+b^2+1)}{2(c^2+b^2)} - 1, \quad y(t) = \frac{c(c^2+b^2-1)}{2(c^2+b^2)}, \quad 0 \leqslant t \leqslant 2\pi, \tag{10}$$

where $c = R\cos(t-\gamma) - d\cos\gamma$, $b = R\sin(t-\gamma) + d\sin\gamma + h$, $R = \sqrt{1+h^2} + d$, and $\gamma = \arctan h$. The parameters d and h characterize the airfoil thickness and curvature. The flow velocity on the airfoil is determined analytically in a parametric form as

$$V_s(t) = \frac{V_\infty R \sin(\beta+\gamma-t) + \Gamma/(2\pi)}{\sqrt{(x'(t))^2 + (y'(t))^2}}, \tag{11}$$

where β is the angle of attack and $\Gamma = -2\pi R V_\infty \sin(\beta+\gamma)$ is the circulation of velocity along the airfoil contour. The numerical value of circulation was determined from the formula [10]

$$\Gamma = \int\limits_C V_s\, ds.$$

The coefficients of the lift force F_y and drag force F_x were calculated using the formulas

$$F_y = -\int\limits_C V_s^2 \sin\alpha(s)\, ds, \qquad F_x = \int\limits_C V_s^2 \cos\alpha(s)\, ds.$$

An additional criterion of the correctness of numerical calculations can be the condition $F_x = 0$, which is the d'Alembert paradox for the case of a perfect fluid.

The coefficient of the force moment M relative to the sharp edge with the coordinates (x_0, y_0) can be calculated using the formula

TABLE 3

β, deg	d	h	Γ	Γ_{200}	E_2	F_x	F_y	M
0	0.2	0.2	−1.503	−1.493	0.004	0.0002	2.978	−3.318
0	0.2	0.5	−3.703	−3.671	0.007	0.001	7.338	−8.108
15	0.2	0	−1.951	−1.935	0.005	0.001	3.868	−5.880
15	0.2	0.2	−3.397	−3.371	0.007	0.002	6.739	−8.886
15	0.2	0.5	−5.494	−5.446	0.009	0.003	10.887	−12.744

$$M(x_0, y_0) = -\int_C V_s^2((x - x_0)\sin\alpha(s) + (y - y_0)\cos\alpha(s))\, ds.$$

The results of the solution can be explained as follows: if $\Gamma < 0$, then $F_y > 0$, and vice versa, if $\Gamma > 0$, then $F_y < 0$. The negative value of the lifting force means that it tries to turn the body relative to the point (x_0, y_0) in the clockwise direction, and the positive value refers to the anticlockwise direction.

The integral Cauchy formula is also valid for the unbounded domain D if the function $W(z)$ vanishes at infinity. In an unbounded liquid flow around the airfoil, the complex potential has the following expansion at infinity:

$$w(z) = \varphi(x, y) + i\psi(x, y) = a + V_\infty e^{-i\beta} z + \frac{\Gamma}{2\pi i}\ln z.$$

Here a is an arbitrary complex constant. The complex-conjugate flow velocity at infinity is determined from the formula

$$W(z) = V_x(x, y) - iV_y(x, y) = V_\infty e^{-i\beta}. \tag{12}$$

With account of (12), Eq. (4) acquires the form

$$W(z_0) = V_\infty e^{-i\beta} + \frac{1}{\varepsilon(z_0)i}\int_C \frac{W(z)}{z - z_0}\, dz.$$

In this case, the motion along the contour is performed in the clockwise direction. In constructing system (8), we have to introduce a term into the known right-side vector $\boldsymbol{F}$ (imaginary or real part of $V_\infty e^{-i\beta}$ to each element of the vector depending on the prescribed boundary condition in the jth node).

Table 3 shows the hydrodynamic characteristics as functions of the angle of attack and airfoil thickness and curvature; Γ and Γ_{200} are the exact and numerical values of circulation, respectively (200 is the number of nodes over the airfoil boundary), $E_2 = \max|V_s^{\text{exact}} - V_s^{\text{num}}| / \max|V_s^{\text{exact}}|$ is the relative error, V_s^{num} is the numerical value of the function obtained using the CMBE, V_s^{exact} is the exact value determined by formula (11), F_x and F_y are the drag and lift coefficients, and M is the lift-force moment relative to the sharp edge of the airfoil. The values of F_x are close to zero, and the relative error of the determined values of V_s is insignificant, which indicates that the method developed is highly accurate.

5. Numerical Results. As an example, we consider two problems: a circulation flow around a circular contour of diameter $d = 0.4H$ and around the Joukowski airfoil ($d = 0.2H$ and $h = 0$) constructed using formulas (10) and reduced proportionally to $l = 0.4H$, where l is the airfoil chord length. The boundary of the domain was approximated by 500 elements (the object was specified by 100 elements, the free boundary by 200 elements, the straight bottom by 150 elements, and the inflow and outflow sectors by 50 elements).

For the Joukowski airfoil, the convergence point of the streamlines is located in the sharp edge; for a circular contour, this point is unknown, and to obtain a unique solution of the problem, one has to specify either velocity circulation on the circular contour or the convergence point. To specify the convergence point, one has to use the Joukowski–Chaplygin condition (2) at this point.

It follows from the analysis of calculation results for a circulation flow around a circular contour and the Joukowski airfoil that the problem has a nonunique solution for Froude numbers close to unity. Figure 2 shows the calculation results for the dependence of the amplitude A on the Froude number Fr in the flow

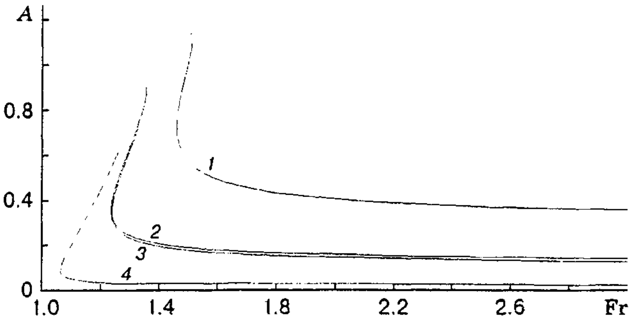

Fig. 2

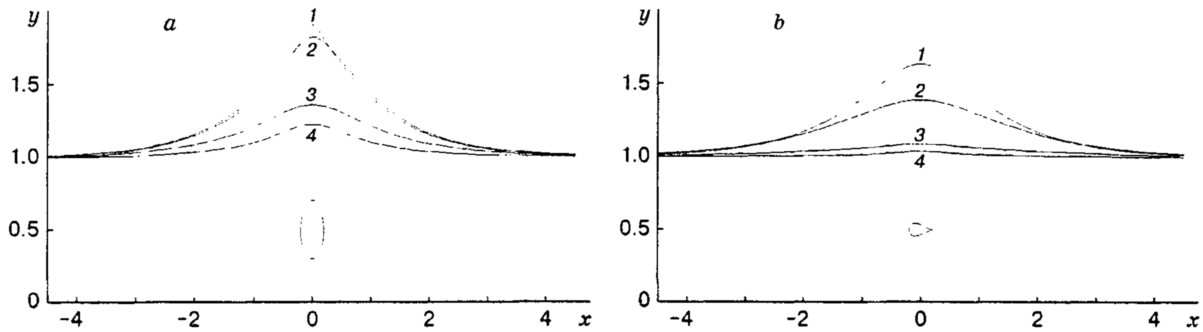

Fig. 3

around a circular contour whose center is located at the point (0; 0.5) [curve 1 corresponds to the convergence point (0.152; 0.37) and curve 2 to (0.2; 0.5)] and the Joukowski airfoil whose sharp edge is located at the point (0.2; 0.5) [curve 3 corresponds to the angle of attack $\beta = 15°$ and curve 4 to $\beta = 0$]. The dashed curve corresponds to the dependence $A_{\max} = \mathrm{Fr}^2/2$ and is the amplitude limit of existence of steady solutions. Curves 3 and 4 do not reach the dashed curve. For a circulation flow around airfoils, apparently, another estimate of the upper boundary is valid for amplitudes for which the problem has a steady solution.

Figure 3 shows the shapes of the free surface in the flow around two bodies: (a) circular contour [convergence point (0.2; 0.5)] for Fr = 1.351 and $A = 0.904$ (curve 1), Fr = 1.346 and $A = 0.818$ (curve 2), Fr = 1.235 and $A = 0.356$ (curve 3), and Fr = 1.346 and $A = 0.22$ (curve 4); (b) Joukowski airfoil ($\beta = 0$) for Fr = 1.264 and $A = 0.634$ (curve 1), Fr = 1.173 and $A = 0.383$ (curve 2), Fr = 1.064 and $A = 0.087$ (curve 3), and Fr = 1.173 and $A = 0.035$ (curve 4). Curves 1 correspond to the wave of the maximum amplitude for which a steady solution was obtained, curves 2 and 4 to an identical Froude number but different amplitudes (these curves demonstrate that the solution is not unique), and curves 3 to the Froude number below which there are no steady solutions.

Figure 4 shows the streamlines of the flow field near the object: (a) circular contour [convergence point (0.2; 0.5)] for Fr = 1.235 and $A = 0.356$; (b) circular contour [convergence point (0.152; 0.37)] for Fr = 1.458 and $A = 0.718$; (c) Joukowski airfoil ($\beta = 0$) for Fr = 1.064 and $A = 0.087$; (d) Joukowski airfoil ($\beta = 15°$) for Fr = 1.237 and $A = 0.355$. The velocity field in the flow region was calculated to construct the streamlines. Good agreement of the flow pattern and the Joukowski condition give indirect evidence of the correctness of the calculations.

Figure 5 shows the hydrodynamic characteristics versus the Froude number. The solid curves correspond to the flow around a circular contour with the convergence point (0.152; 0.37) and the dashed curves to the flow around the Joukowski airfoil with an angle of attack $\beta = 15°$. The plots for two other objects are not presented because their hydrodynamic characteristics are too small.

Conclusions. An effective numerical method for calculation of a nonlinear steady problem of the

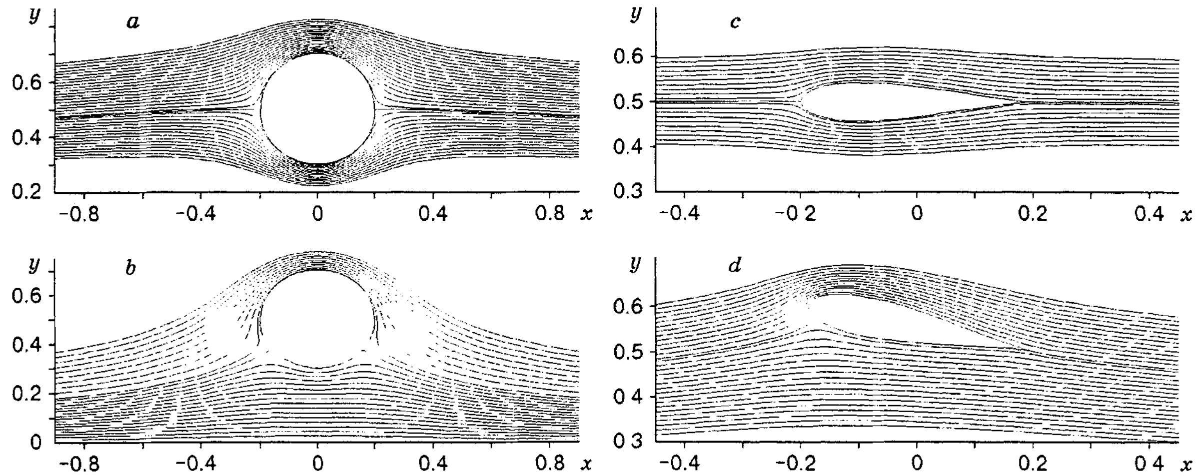

Fig. 4

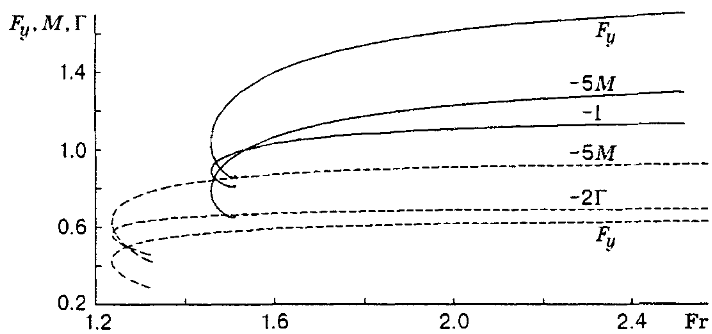

Fig. 5

circulation flow around an airfoil under the free surface of a perfect ponderable liquid is developed in the present paper. To satisfy the Joukowski–Chaplygin condition, an approach different from those in the cited papers on this topic was used at the sharp edge. To solve the problem, the CMBE was modified, and the well-proven algorithm of constructing the free boundary [8, 9] was used. In the circulation flow around a circular contour and the Joukowski airfoil, the solution is not unique for Froude numbers close to unity. Since this problem is multiparametric, the effect of all the parameters of the problem on the hydrodynamic characteristics of the airfoil will be studied later in more detail.

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